EE 508
Lecture 31

Leapfrog Networks

Laboratory Procedures were A Major Problem in the Laboratory Yesterday !

Engineer's Notebook – A Design Assessment Tool

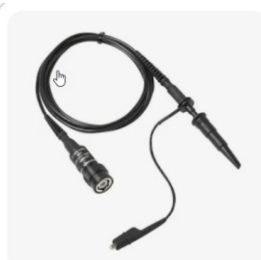
By Todd R. Kelley

https://www.purdue.edu/trails/wp-content/uploads/2019/06/23.Engineer_s_notebook_TET-article.pdf

“Scope” Probes

What is an oscilloscope probe?

An oscilloscope probe is a device that makes a physical and electrical connection between a test point or signal source and an oscilloscope. Depending on your measurement needs, this connection can be made with something as simple as a length of wire or with something as sophisticated as an active differential probe. Essentially, a probe is some sort of device or network that connects the signal source to the input of the



Tektronix Oscilloscope Probe Probe Type Passive TPP0101

$67.00

Mouser Electro… , 5+ stores

"Scope" Probes

What is an oscilloscope probe?

Why are scope probes used?

What is the major reason scope probes that are more complicated than a piece of wire are used in our laboratories?

“Scope” Probes

Why are scope probes used?

Are there disadvantages for using a commercial scope probe when they are not essential?

Should you be using commercial scope probes in our laboratories?

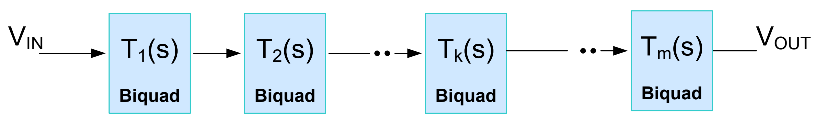
“Scope” Probes

How does a 1X or 10X probe operate?

Review from last lecture

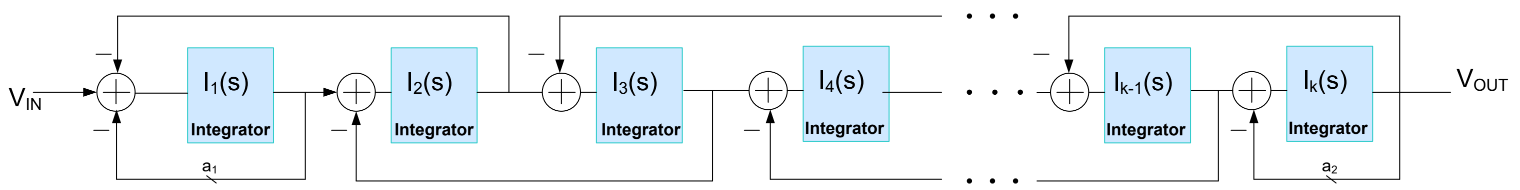
Filter Design/Synthesis Approaches

Cascaded Biquads

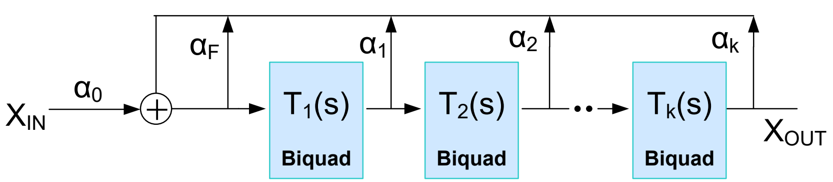


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog

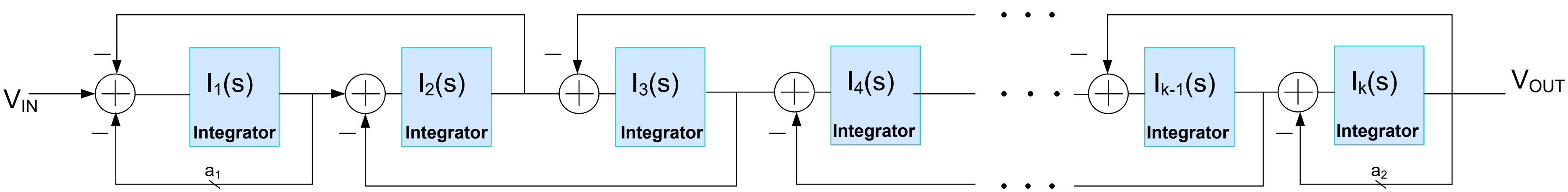


Multiple-loop Feedback – One type shown



Review from last lecture

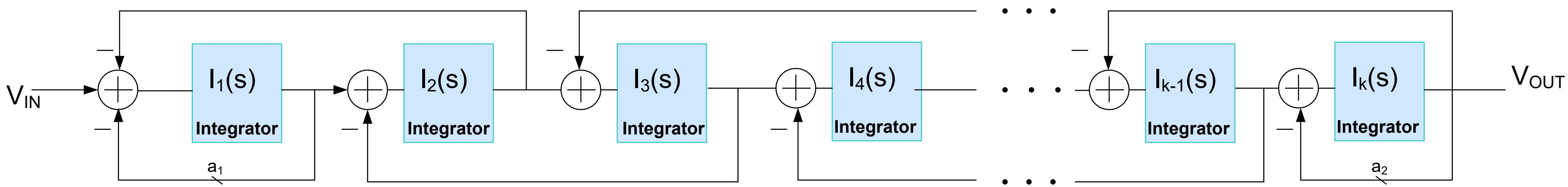
Leapfrog Filters



Introduced by Girling and Good, Wireless World, 1970

This structure has some very attractive properties and is widely used though the real benefits and limitations of the structure are often not articulated

Leapfrog Filters

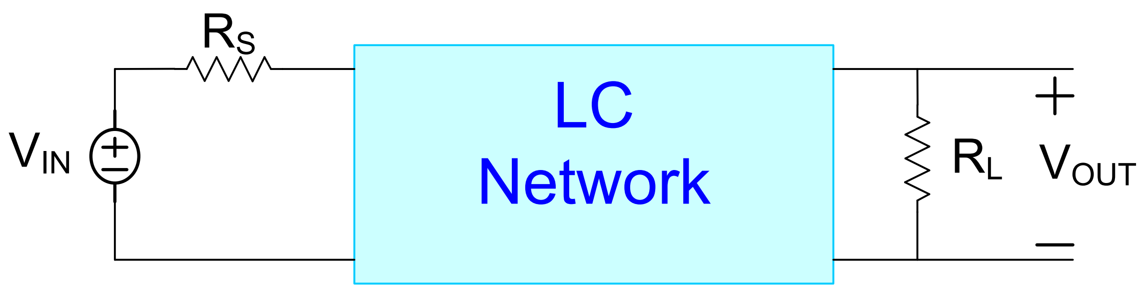


Observation: This structure appears to be dramatically different than anything else ever reported and it is not intuitive why this structure would serve as a filter, much less, have some unique and very attractive properties

To understand how the structure arose, why it has attractive properties, and to identify limitations, some mathematical background is necessary

Review from last lecture

Background Information for Leapfrog Filters



Assume the impedance R_S is fixed

Theorem 1: If the LC network delivers maximum power to the load at a frequency ω, then

$$S_x^{|T(j\omega)|} = 0$$

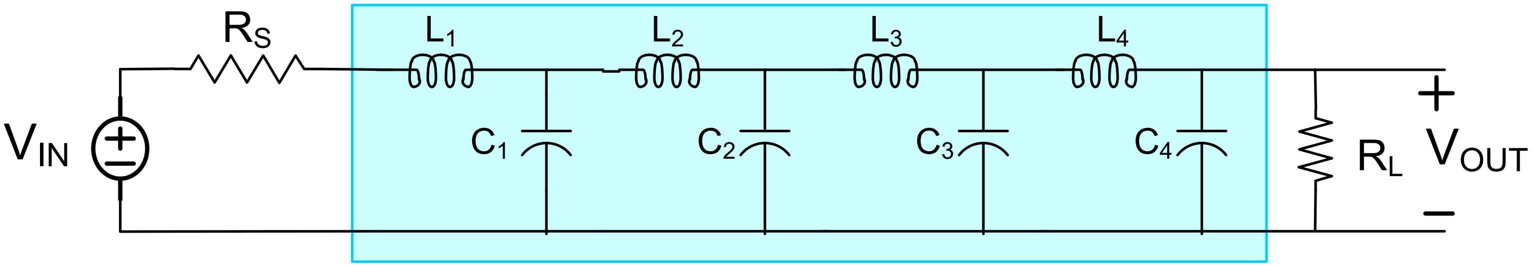
for any circuit element in the system except for $x = R_L$

This theorem will be easy to prove after we prove the following theorem:

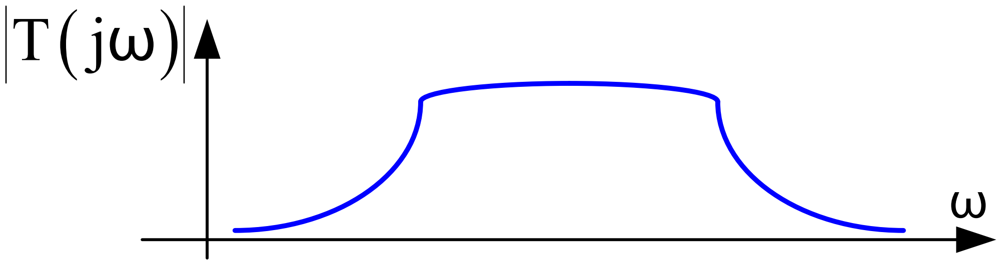
Review from last lecture

Implications of Theorem 1

Many passive LC filters such as that shown below exist that have near maximum power transfer in the passband



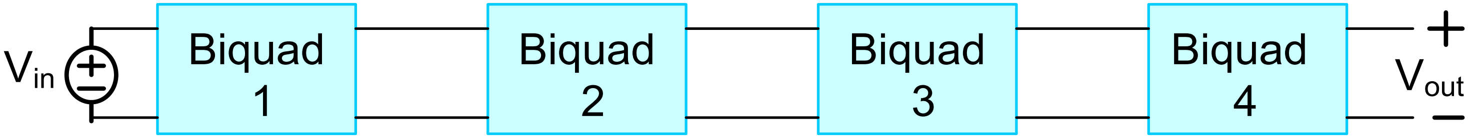
If a component in the LC network changes a little, there is little change in the passband gain characteristics (depicted as bandpass)



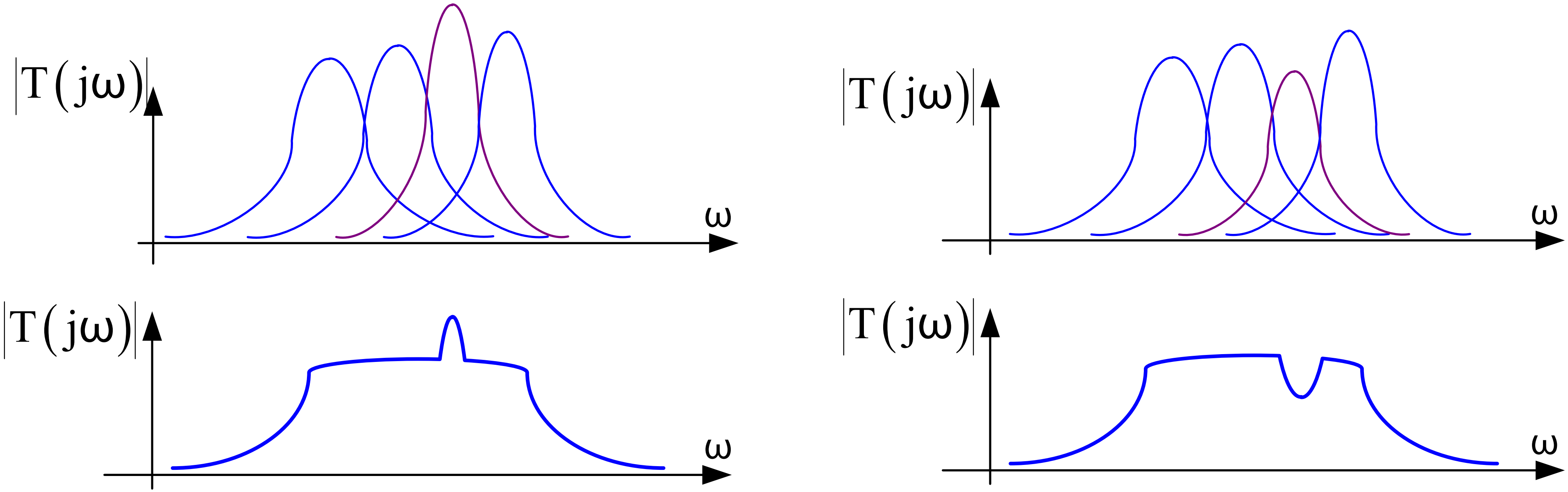
$$S_x^{|T(j\omega)|} \simeq 0$$ in passband

Review from last lecture

Implications of Theorem 1



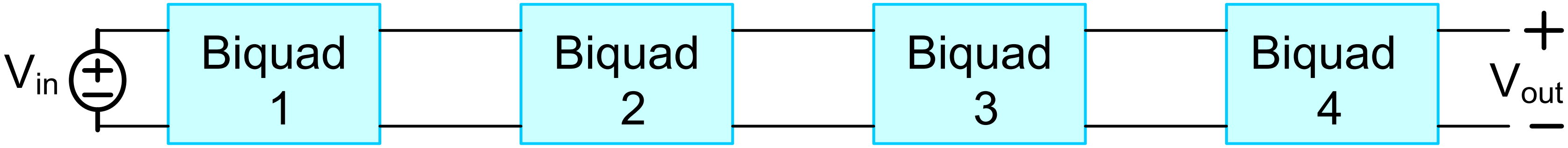
If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)

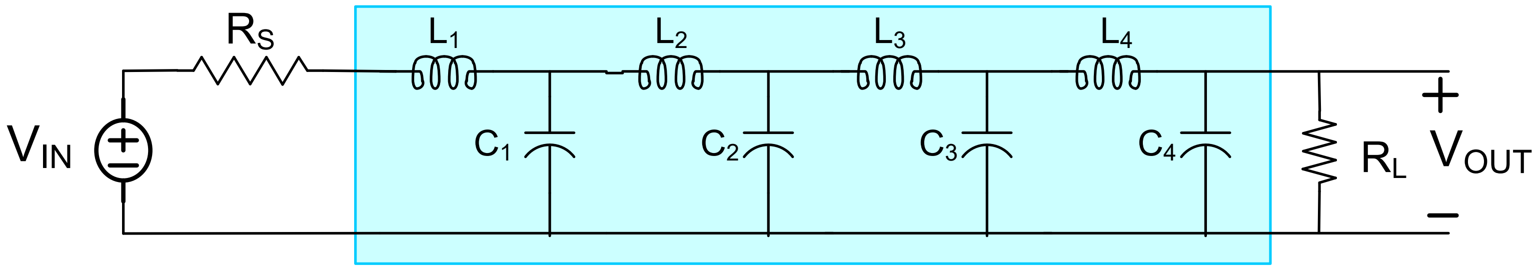


$$S_x^{|T(j\omega)|} \neq 0$$ in passband

Review from last lecture

Implications of Theorem 1

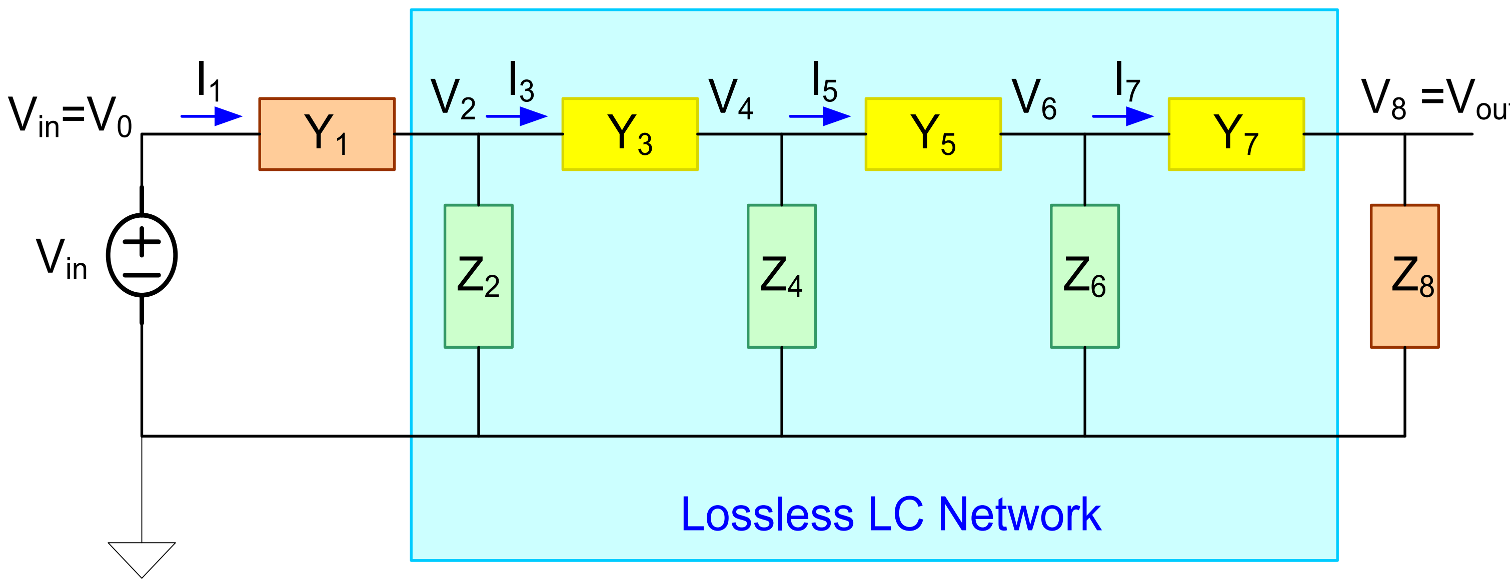




Good doubly-terminated LC networks often much less sensitive to most component values in the passband than are cascaded biquads !

This is a major advantage of the LC networks but can not be applied practically in most integrated applications or even in pc-board based designs

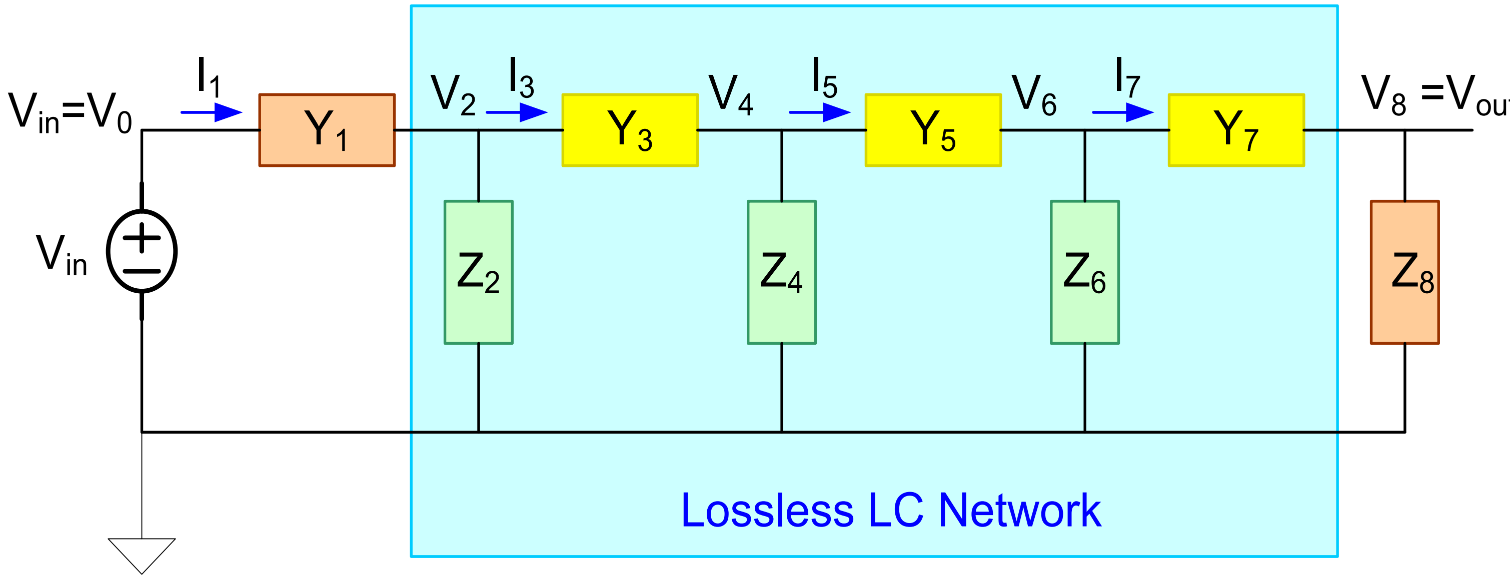
Doubly-terminated Ladder Network with Low Passband Sensitivities



For components in the LC Network observe

$$Y_k = \frac{1}{sL_k} \qquad\qquad Z_k = \frac{1}{sC_k}$$

Doubly-terminated Ladder Network with Low Passband Sensitivities



$$
\left.\begin{aligned}
I_1 &= (V_0 - V_2)Y_1 \\
V_2 &= (I_1 - I_3)Z_2 \\
I_3 &= (V_2 - V_4)Y_3 \\
V_4 &= (I_3 - I_5)Z_4 \\
I_5 &= (V_4 - V_6)Y_5 \\
V_6 &= (I_5 - I_7)Z_6 \\
I_7 &= (V_6 - V_8)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}\right\}
$$

Complete set of independent equations that characterize this filter

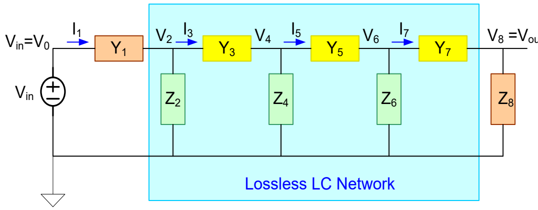
Solution of this set of equations is tedious

All sensitivity properties of this circuit are inherently embedded in these equations!

Consider now only the set of equations and disassociate them from the circuit from where they came

$$\left.\begin{aligned}
I_1 &= (V_0 - V_2)Y_1 \\
V_2 &= (I_1 - I_3)Z_2 \\
I_3 &= (V_2 - V_4)Y_3 \\
V_4 &= (I_3 - I_5)Z_4 \\
I_5 &= (V_4 - V_6)Y_5 \\
V_6 &= (I_5 - I_7)Z_6 \\
I_7 &= (V_6 - V_8)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}\right\}$$

Make the associations



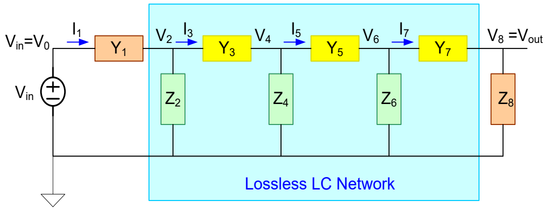
$$I_1 = V_1'$$
$$I_3 = V_3'$$
$$I_5 = V_5'$$
$$I_7 = V_7'$$

Rewrite the equations as

$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)Y_1 \\
V_2 &= (V_1' - V_3')Z_2 \\
V_3' &= (V_2 - V_4)Y_3 \\
V_4 &= (V_3' - V_5')Z_4 \\
V_5' &= (V_4 - V_6)Y_5 \\
V_6 &= (V_5' - V_7')Z_6 \\
V_7' &= (V_6 - V_8)Y_7 \\
V_8 &= V_7' Z_8
\end{aligned}\right\}$$

This association is nothing more than a renaming of variables so all sensitivities WRT Y's and Z's will remain unchanged!

Consider now only the set of equations and disassociate them from the circuit from where they came



$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)Y_1 \\
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V_4 &= (V_3' - V_5')Z_4 \\
V_5' &= (V_4 - V_6)Y_5 \\
V_6 &= (V_5' - V_7')Z_6 \\
V_7' &= (V_6 - V_8)Y_7 \\
V_8 &= V_7' Z_8
\end{aligned}\right\}$$

For the LC filter, recall

$$Y_k = \frac{1}{sL_k} \qquad Z_k = \frac{1}{sC_k}$$

And the source and load termination relationships were

$$Y_1 = \frac{1}{R_1} \qquad Z_8 = R_8$$

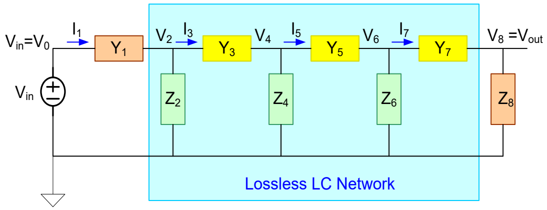
These can be written as

$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & V_5' &= (V_4 - V_6)\frac{1}{sL_5} \\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & V_6 &= (V_5' - V_7')\frac{1}{sC_6} \\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & V_7' &= (V_6 - V_8)\frac{1}{sL_7} \\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}\right\}$$

Observe that in the new parameter domain the equations all look like integrator functions if the primed and unprimed variables are all voltages !

Consider now only the set of equations and disassociate them from the circuit from where they came

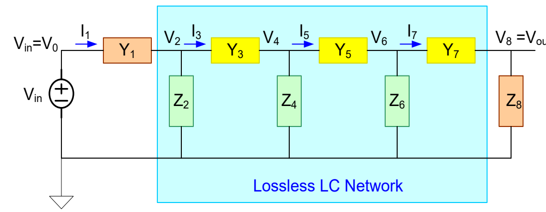
$$\left.\begin{aligned}
V_1' &= \left(V_0 - V_2\right)\frac{1}{R_1} & V_5' &= \left(V_4 - V_6\right)\frac{1}{sL_5} \\
V_2 &= \left(V_1' - V_3'\right)\frac{1}{sC_2} & V_6 &= \left(V_5' - V_7'\right)\frac{1}{sC_6} \\
V_3' &= \left(V_2 - V_4\right)\frac{1}{sL_3} & V_7' &= \left(V_6 - V_8\right)\frac{1}{sL_7} \\
V_4 &= \left(V_3' - V_5'\right)\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}\right\}$$



Observe that in the new parameter domain the equations all look like integrator functions if the primed and unprimed variables are all voltages !

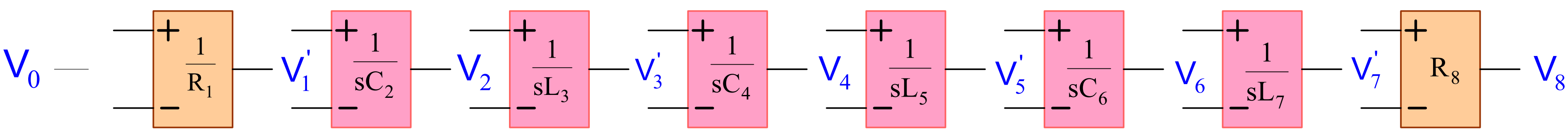
If any circuit is characterized by these equations, the sensitivities to the integrator gains will be identical to the sensitiviies of the original circuit to the Ls and Cs !

Consider now only the set of equations and disassociate them from the circuit from where they came



$$
\left.
\begin{array}{ll}
V_1' = \left(V_0 - V_2\right)\dfrac{1}{R_1} & V_5' = \left(V_4 - V_6\right)\dfrac{1}{sL_5} \\
V_2 = \left(V_1' - V_3'\right)\dfrac{1}{sC_2} & V_6 = \left(V_5' - V_7'\right)\dfrac{1}{sC_6} \\
V_3' = \left(V_2 - V_4\right)\dfrac{1}{sL_3} & V_7' = \left(V_6 - V_8\right)\dfrac{1}{sL_7} \\
V_4 = \left(V_3' - V_5'\right)\dfrac{1}{sC_4} & V_8 = V_7' R_8
\end{array}
\right\}
$$

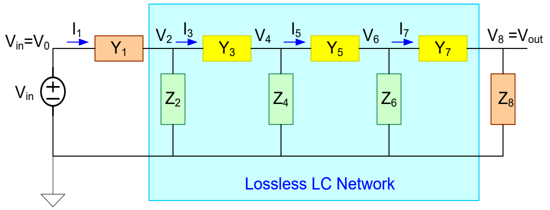
Each equation corresponds to either an integrator or summer with the output voltage output variables and the gain indicated (don’t worry about the units)



$$V_0 = V_{in}$$

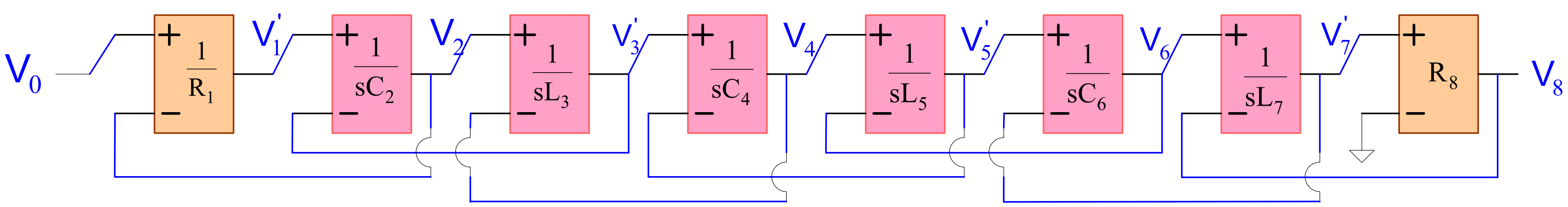
$$V_8 = V_{out}$$

Consider now only the set of equations and disassociate them from the circuit from where they came

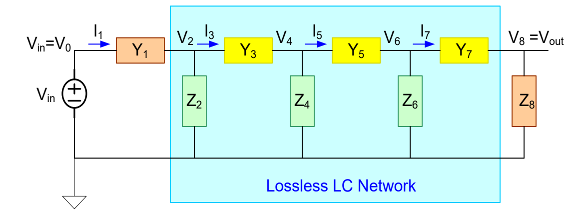


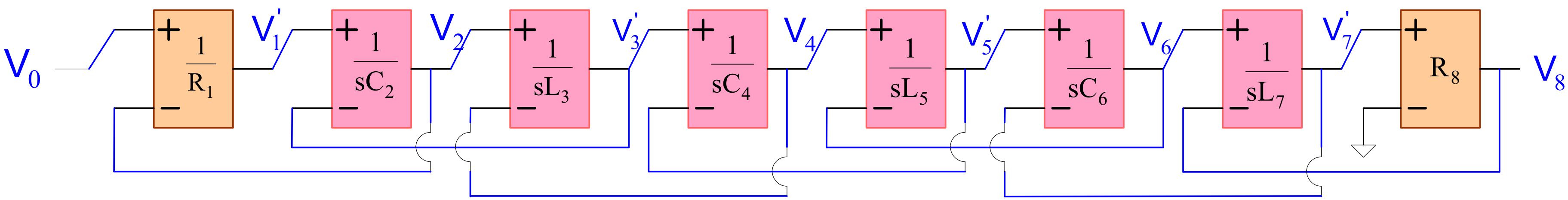
$$
\left.
\begin{array}{ll}
V_1' = (V_0 - V_2)\dfrac{1}{R_1} & V_5' = (V_4 - V_6)\dfrac{1}{sL_5} \\
V_2 = (V_1' - V_3')\dfrac{1}{sC_2} & V_6 = (V_5' - V_7')\dfrac{1}{sC_6} \\
V_3' = (V_2 - V_4)\dfrac{1}{sL_3} & V_7' = (V_6 - V_8)\dfrac{1}{sL_7} \\
V_4 = (V_3' - V_5')\dfrac{1}{sC_4} & V_8 = V_7' R_8
\end{array}
\right\}
$$

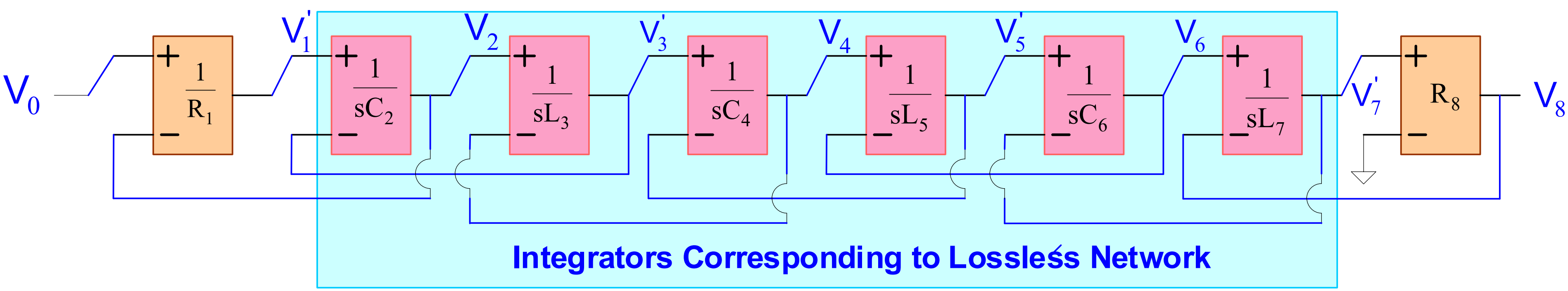
The interconnections that complete each equation can now be added



Consider now only the set of equations and disassociate them from the circuit from where they came



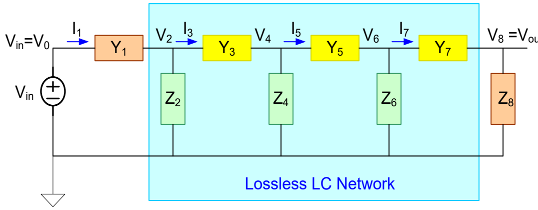


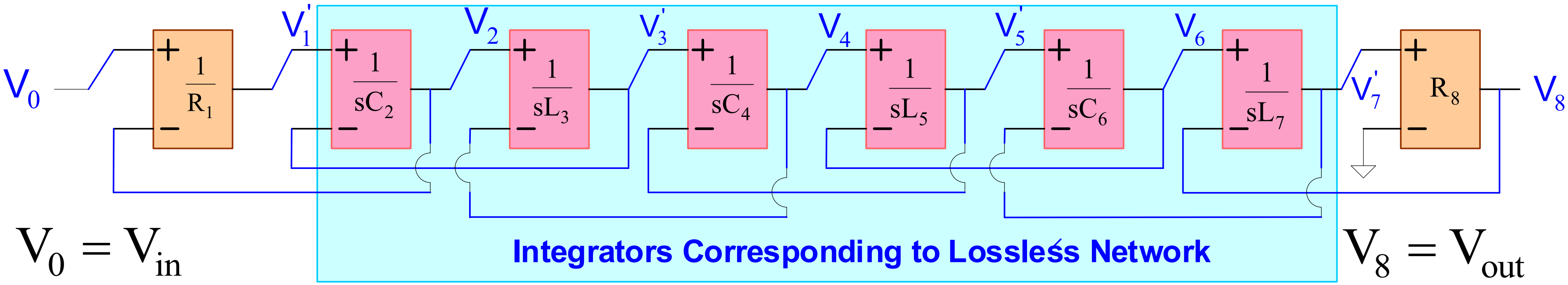


$$V_0 = V_{in}$$

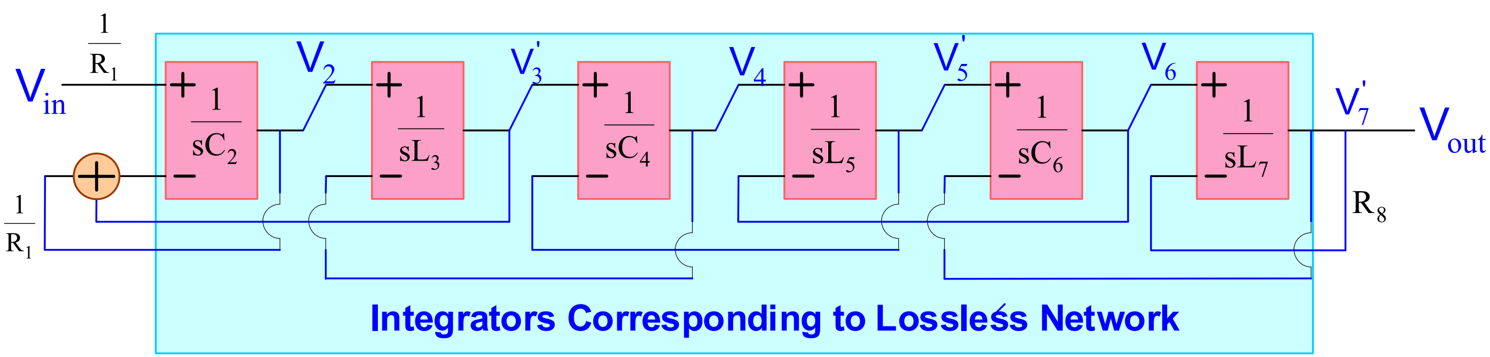
$$V_8 = V_{out}$$

The Leapfrog Configuration

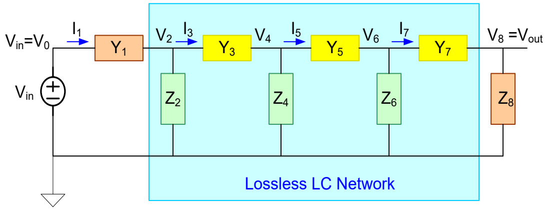


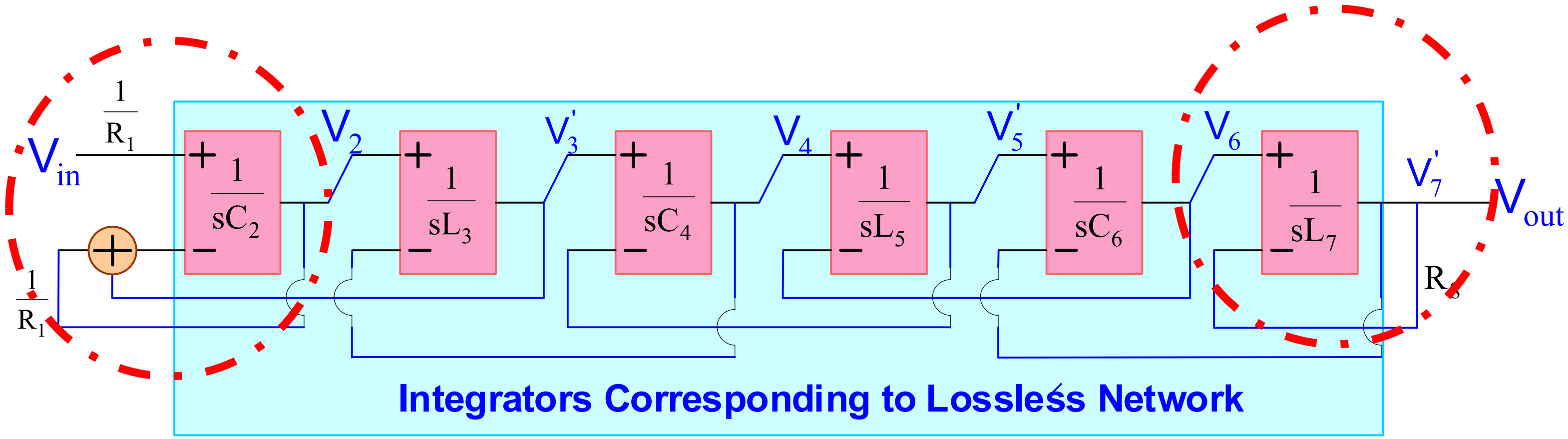


Input summing and weighting can occur at input to the first integrator
The difference between V_8 and V'_7 is only a scale factor that does not affect shape, and the weighting on the Vin input also does not affect shape, thus



The Leapfrog Configuration

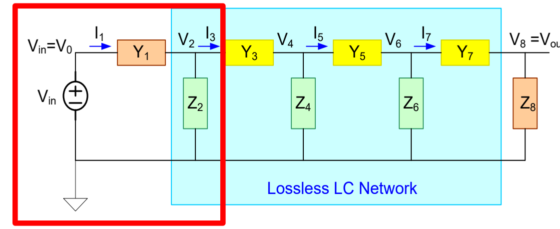


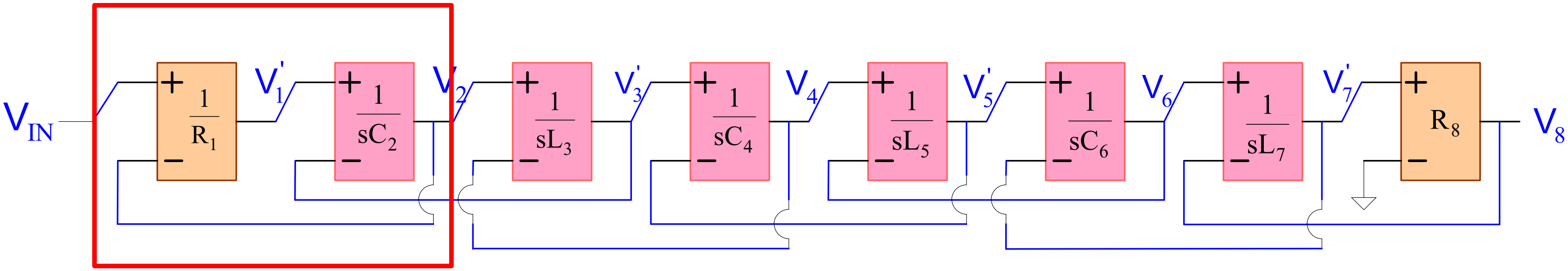


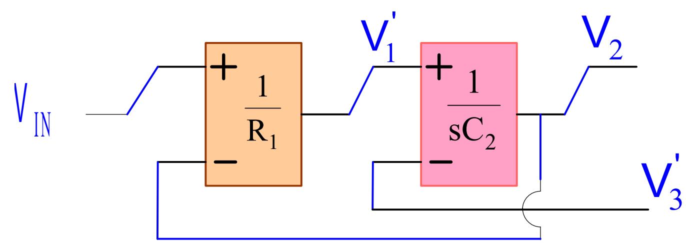
The terminations on both sides have local feedback around an integrator which can be alternately viewed as a lossy integrator

Could redraw the structure as a cascade of internal lossless integrators with terminations that are lossy integrators but since there are so many different ways to implement the integrators and summers, we will not attempt to make that association in the block diagram form but in most practical applications a lossy integrator is often used on the input or the output or both

Consider the first two stages:





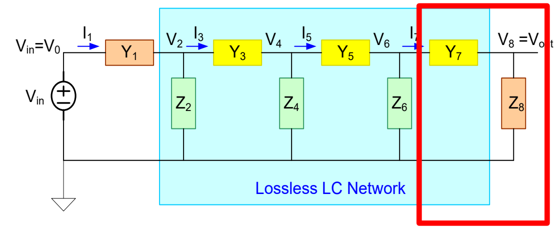


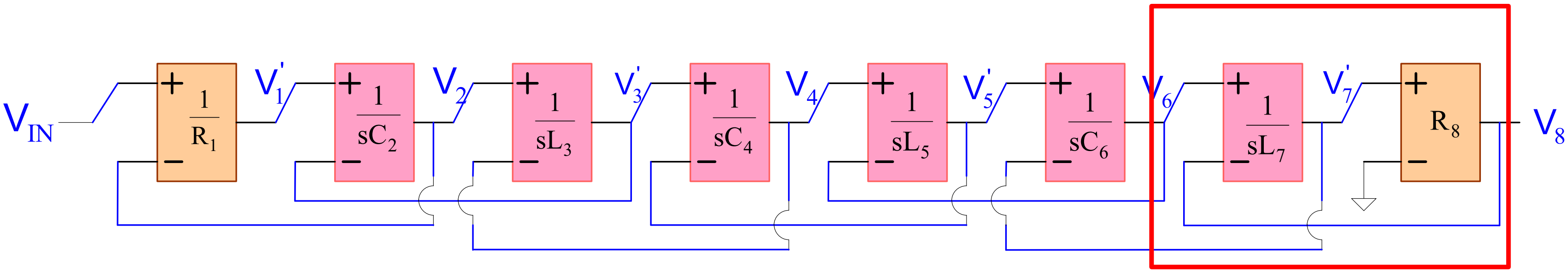
$$\left.\begin{array}{l} V_1' = (V_0 - V_2)\dfrac{1}{R_1} \\[2ex] V_2 = (V_1' - V_3')\dfrac{1}{sC_2} \end{array}\right\} \quad V_2 = \left((V_0 - V_2)\frac{1}{R_1} - V_3'\right)\frac{1}{sC_2}$$

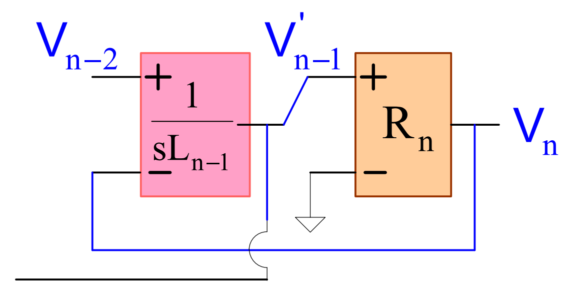
$$V_2 = V_{IN}\left(\frac{1}{1+R_1C_2s}\right) - V_3'\left(\frac{R_1}{1+R_1C_2s}\right)$$

These two blocks act as a single summing lossy integrator block with loss factor R_1

Consider the last two stages:







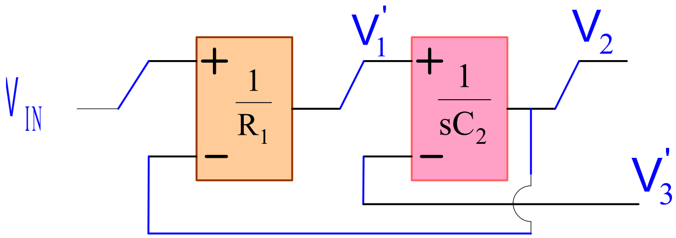
$$\left.\begin{aligned} V_{n-1}' &= (V_{n-2} - V_n)\frac{1}{sL_{n-1}} \\ V_n &= V_{n-1}' R_n \end{aligned}\right\}$$

$$V_n = (V_{n-2} - V_n)\frac{1}{sL_{n-1}} R_n$$

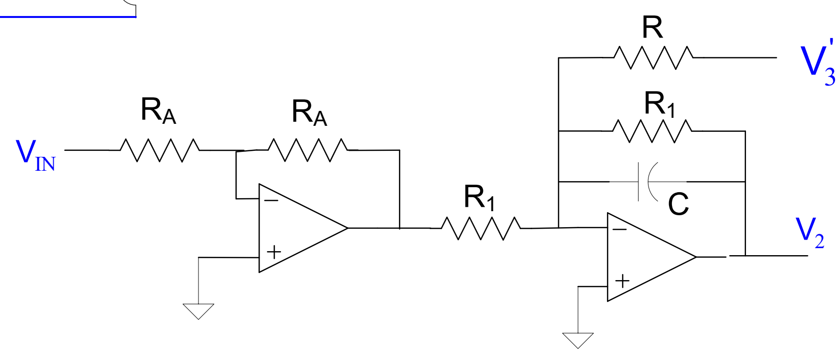
$$V_n = V_{n-2}\left(\frac{R_n}{sL_{n-1} + R_n}\right)$$

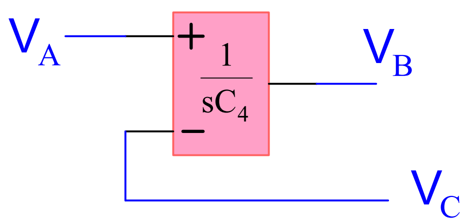
These two blocks act as a lossy integrator block with loss factor R_n

Implementation with Miller Integrators:

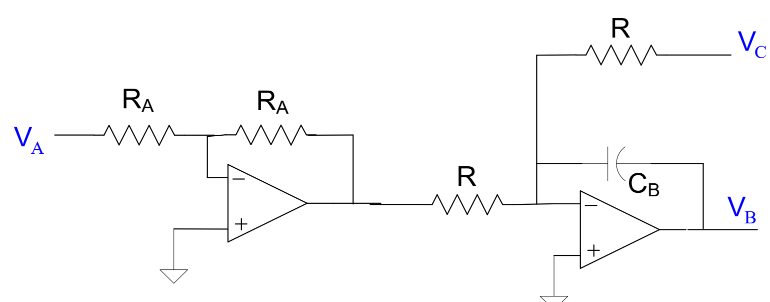


$$V_2 = V_{IN}\left(\frac{1}{1+R_1C_2s}\right) - V_3'\left(\frac{R_1}{1+R_1C_2s}\right)$$



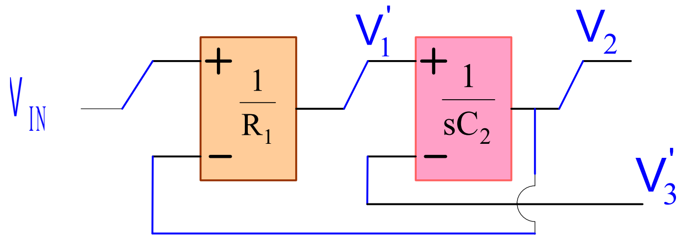


$$V_B = V_A\left(\frac{1}{RC_Bs}\right) - V_C\left(\frac{1}{1+RC_Bs}\right)$$

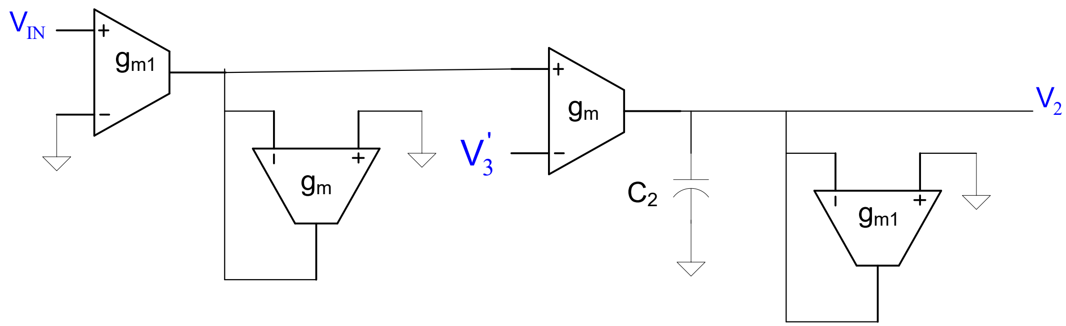


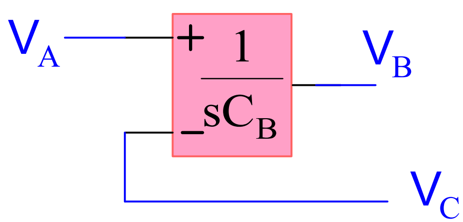
Can fix either R or C on each stage

Implementation with OTA-C Integrators:

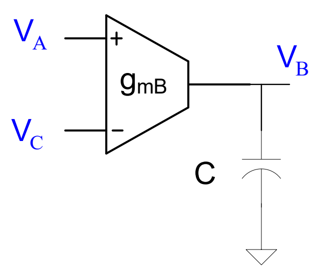


$$V_2 = V_{IN}\left(\frac{1}{1+R_1C_2s}\right) - V_3'\left(\frac{R_1}{1+R_1C_2s}\right)$$



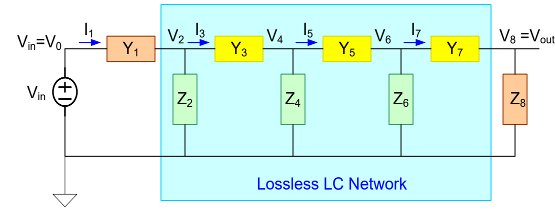


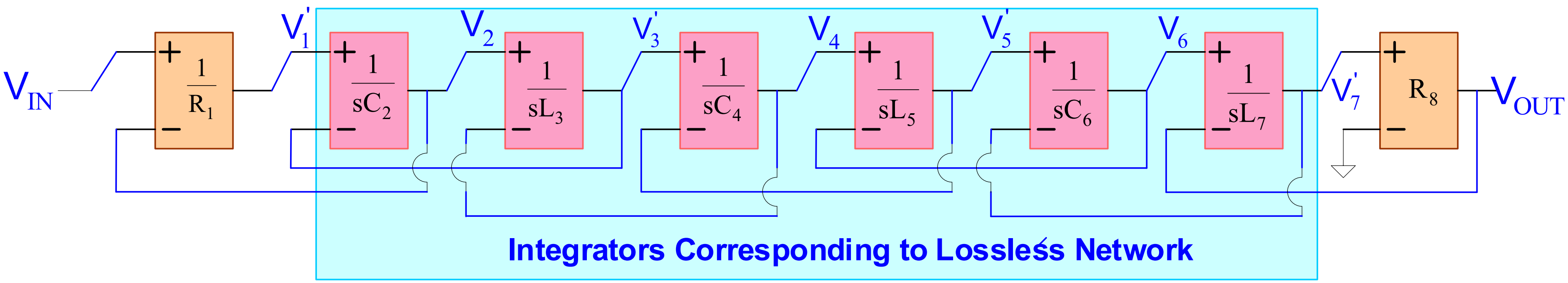
$$V_B = V_A\left(\frac{1}{RC_Bs}\right) - V_C\left(\frac{1}{1+RC_Bs}\right)$$



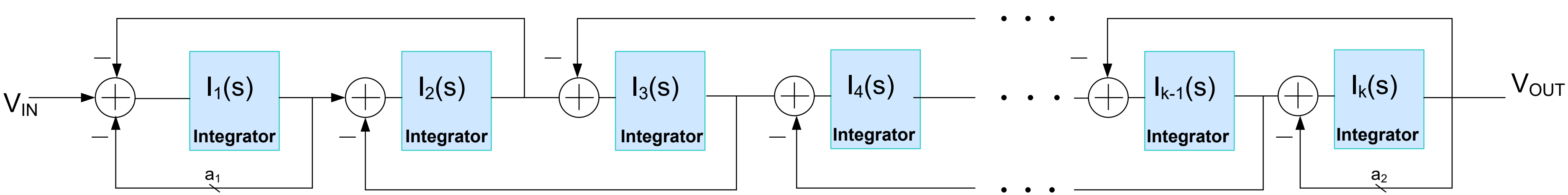
Can fix either g_m or C on each stage

The Leapfrog Configuration



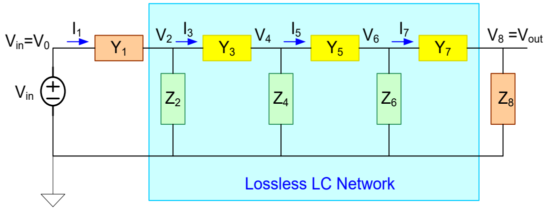


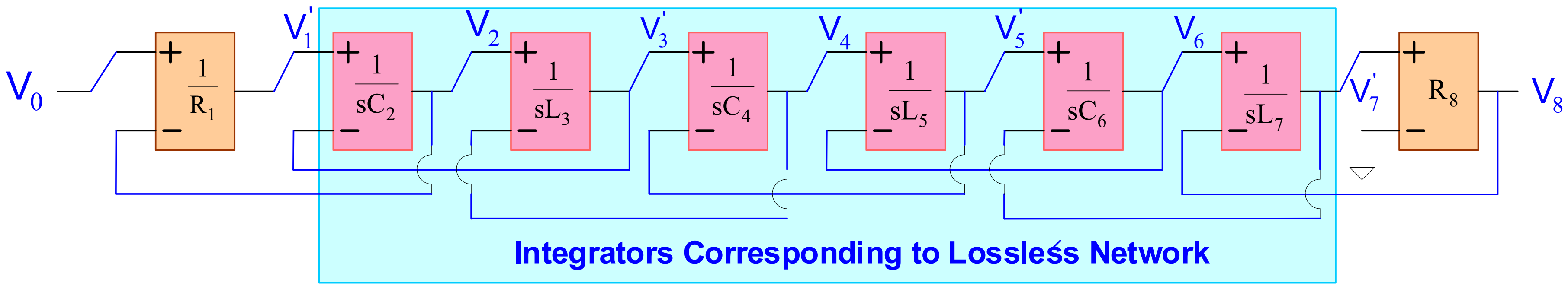
In the general case, this can be redrawn as shown below



Note the first and last integrators become lossy because of the local feedback

The Leapfrog Configuration





The passive prototype filter from which the leapfrog was designed has all shunt capacitors and all series inductors and is thus lowpass.

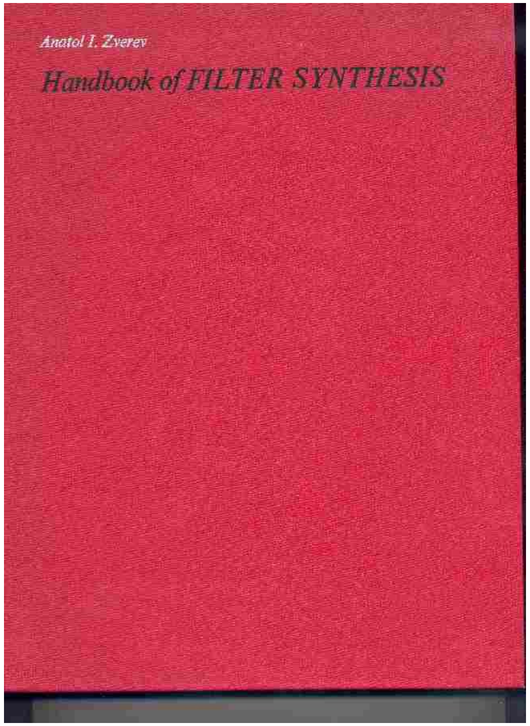
The resultant leapfrog filter has the same transfer function and is thus lowpass

The Passive Prototypes with Maximum Power Transfer in Passband

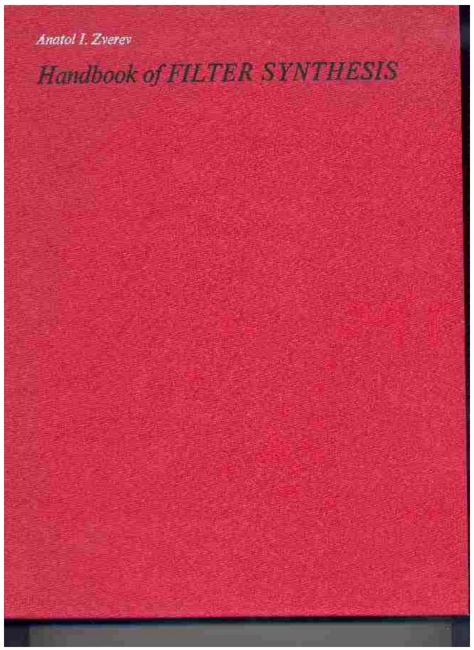
Doubly-terminated LC filters with near maximum power transfer in the passband were developed from the 30's to the 60's

Seldom discussed in current texts but older texts and occasionally software tools provide the passive structures needed to synthesize leapfrog networks

One good book is that by Zverev

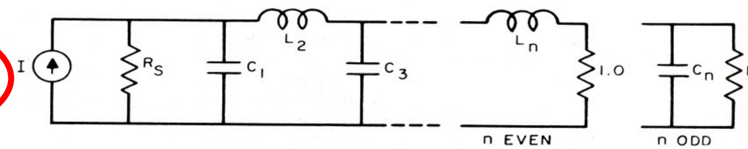


The Passive Prototypes with Maximum Power Transfer in Passband

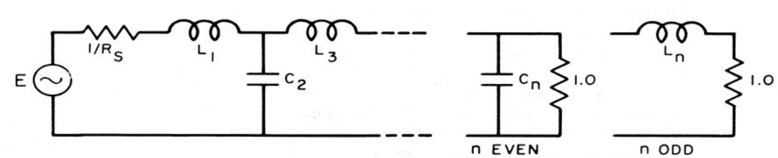


LOW PASS ELEMENT VALUES

BUTTERWORTH RESPONSE



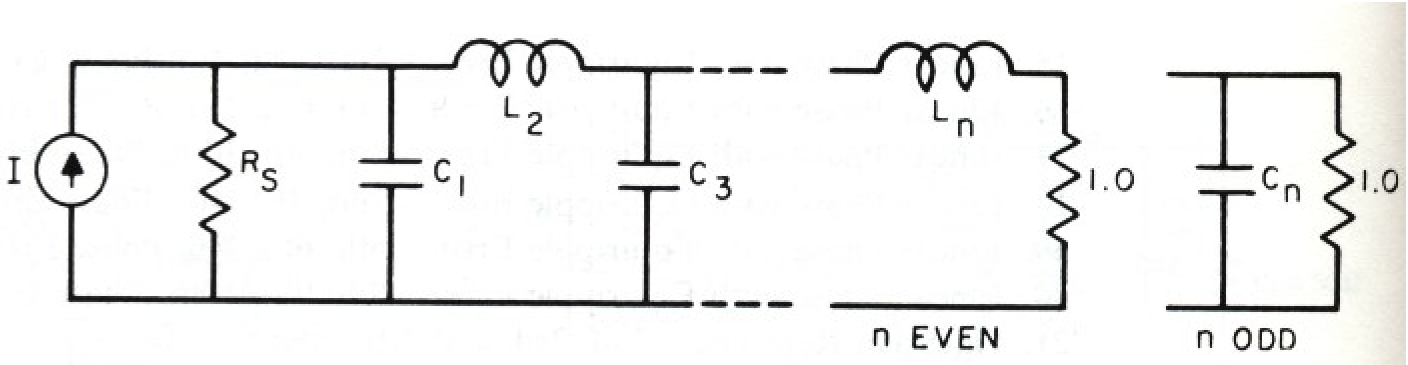
n	R_s	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.8352		
	1.2500	0.8485	2.1213		
	1.4286	0.6971	2.4387		
	1.6667	0.5657	2.8284		
	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8138		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	0.9000	0.8082	1.6332	1.5994	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3882	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8618	1.2913	2.1752
	1.6667	0.2690	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.3381
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_s$	L_1	C_2	L_3	C_4



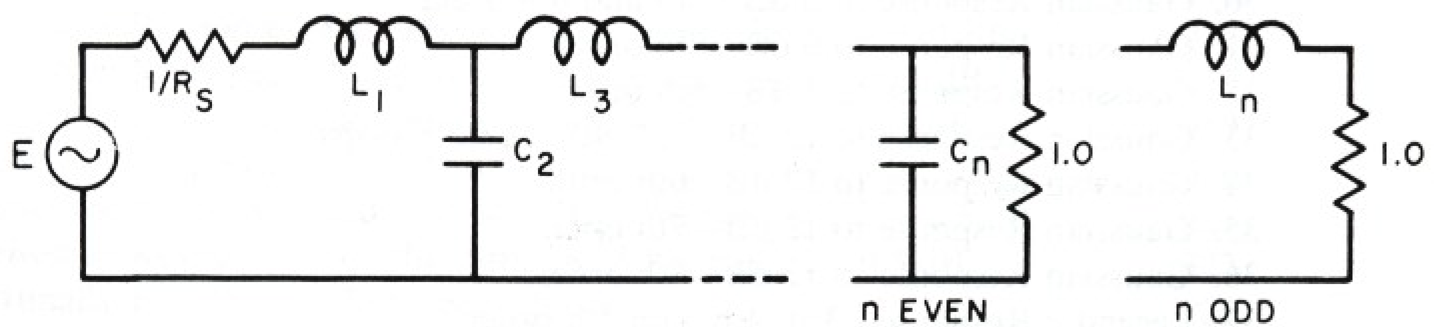
Must start with correct filter type:
(e.g. BW, CC, Cauer)

The Passive Prototypes with Maximum Power Transfer in Passband

The Butterworth Low-Pass Filters



First element is capacitor (appear from top to bottom in table)



First element is inductor (appear from bottom to top in table)

Can do Thevenin-Norton Transformations

The Passive Prototypes with Maximum Power Transfer in Passband

Normalized so R_L=1

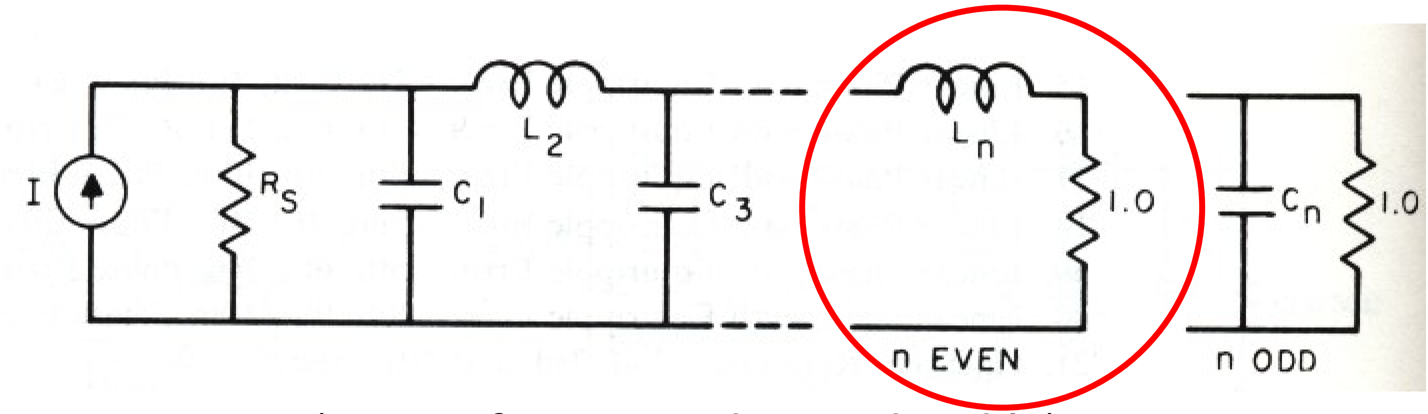
n	R_s	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.8352		
	1.2500	0.8485	2.1213		
	1.4286	0.6971	2.4387		
	1.6667	0.5657	2.8284		
	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8138		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	0.9000	0.8082	1.6332	1.5994	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3882	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8618	1.2913	2.1752
	1.6667	0.2690	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.3381
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_s$	L_1	C_2	L_3	C_4

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	INF.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	INF.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	INF.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

Example:

Design a 6th-order BW lowpass Leapfrog filter with equal source and load terminations, and with a 3dB band edge of 4KHz.

Start with the normalized BW lowpass filter



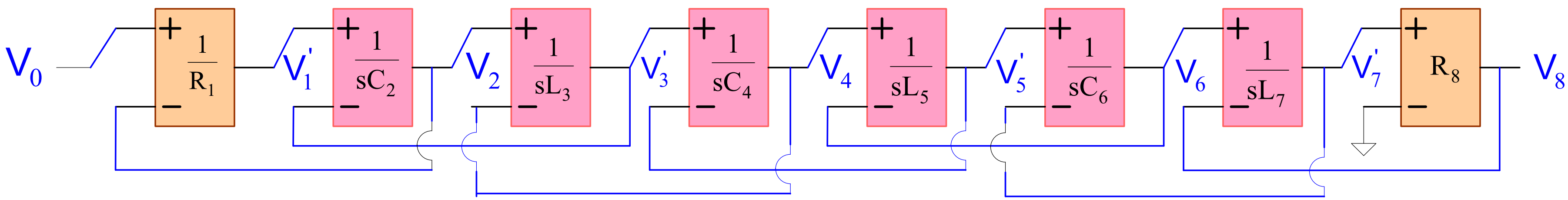
(appear from top to bottom in table)

Do Norton to Thevenin transformation at input

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	INF.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	INF.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	INF.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

$R_s=1$, $C_1=.5176$, $L_2=1.414$, $C_3=1.939$, $L_4=1.9319$, $C_5=1.4142$, $L_6=0.5176$

Note index differs by 1 from that used for Leapfrog formulation



Labeled voltages are single-ended voltages at "+" inputs to the integrators

Changing the index notation:

R_1=1, C_2=.5176, L_3=1.414, C_4=1.939, L_5=1.9319, C_6=1.4142, L_7=0.5176

Implement in the technology of choice

Combine loss on input and output integrators to eliminate two stages

Do frequency denormalization to obtain band-edge at 4KHz

Do impedance scaling to obtain acceptable component values

Bandpass Leapfrog Structures

Consider lowpass to bandpass transformations

Un-normalized

$$s_n \rightarrow \frac{s^2 + \omega_0^2}{sBW}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$

$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

Normalized

$$s_n \rightarrow \frac{s^2 + 1}{sBW_n}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW_n}{s^2 + 1}$$

$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW_n}{s^2 + s\alpha BW_n + 1}$$

Bandpass Leapfrog Structures

Bandpass Leapfrog Structure obtained by replacing integrators by the corresponding transformed block

Zero sensitivity to parameters in the transformed blocks will be retained at the image frequencies of the bandpass filter

$$s_n \rightarrow \frac{s^2 + \omega_0^2}{s\mathrm{BW}}$$

$$\frac{1}{s_n} \rightarrow \frac{s\mathrm{BW}}{s^2 + \omega_0^2}$$

Integrators map to bandpass biquads with infinite Q

$$\frac{1}{s_n + \alpha} \rightarrow \frac{s\mathrm{BW}}{s^2 + s\alpha\mathrm{BW} + \omega_0^2}$$

Lossy integrators map to bandpass biquads with finite Q

Bandpass Leapfrog Structures

Bandpass Leapfrog Structure obtained by replacing integrators by the corresponding transformed block

Zero sensitivity to parameters in the transformed blocks will be retained at the image frequencies of the bandpass filter

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2+\omega_0^2}$$

Integrators map to bandpass biquads with infinite Q

$$\frac{1}{s_n+\alpha} \rightarrow \frac{sBW}{s^2+s\alpha BW+\omega_0^2}$$

Lossy integrators map to bandpass biquads with finite Q

Invariably the resistance spread or the capacitance spread increases with Q

- Does this imply that the area will get very large if Q gets large?
- But what about infinite Q?
- Will infinite Q biquads be unstable?
- Is this a problem ?

Stay Safe and Stay Healthy !

End of Lecture 31